

The Product Metric

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December 2, 2017

The *natural* metric for a product of finite sequence of metric spaces is called the *product metric*. We define what this is mention why it is the most natural of other choices.

If $(X_1, d_1), (X_2, d_2), \dots, (X_n, d_n)$ is a finite sequence of metric spaces and N is the *Euclidean norm*, then:

$$(X_1 \times X_2 \times \dots \times X_n, N(d_1, \dots, d_n))$$

is a metric space called the **product metric**. This is defined by

$$N(d_1, \dots, d_n)((x_1, \dots, x_n), (y_1, \dots, y_n)) = N(d_1(x_1, y_1), \dots, d_n(x_n, y_n))$$

This actually induces the *product topology* making it the most natural choice for a product metric.