P2. Let $\mathbf{MAX}(L) = \{x \in L : \text{ for any } y \in \Sigma^*,\ y \neq \epsilon, \text{ then } xy \notin L\}$. This is an operation on a language L. We show that the class of regular languages is closed under this operation and we go about doing this by supposing some DFSA M exists and accepts L, then we construct another FSA M' that accepts $\mathbf{MAX}(L)$.
Suppose a DFSA $M = (Q, \Sigma, \delta, s, F)$ exists and accepts L. Then, let F' be the set of accepting states in M such that we *cannot* reach any other accepting state of M by following some sequence of transitions i.e.

$$F' = \{x \in F : \text{ for all } y \in \Sigma^*,\ \text{if } y \neq \epsilon,\ \text{then } \delta^*(x, y) \notin F\}$$

Then, consider the DFSA $M' = (Q, \Sigma, \delta, s, F')$, then we know the following is true:

$$\begin{aligned} x \in \mathcal{L}(M') &= \delta^*(s, x) \in F' \\ &= \delta^*(s, x) \in F, \text{ and for all } y \in \Sigma^*,\ \text{if } y \neq \epsilon,\ \text{then } \delta^*(\delta^*(s, x), y) \notin F \end{aligned}$$

Then, we can simplify $\delta^*(\delta^*(s, x), y)$ to $\delta^*(s, xy)$ so that $x \in \mathcal{L}(M')$ if and only if $x \in L$, and for all $y \in \Sigma^*$, if $y \neq \epsilon$, then $xy \notin L$. However, this is simply the definition of our operation so, we get that $x \in \mathcal{L}(M')$ if and only if $x \in \mathbf{MAX}(L)$.